



# **Model for determining the Norwegian deposit guarantee fund liabilities - Version III: Technical report**

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## Norwegian Computing Center

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### **Abstract**

The Norwegian Banks' Guarantee Fund is the deposit insurer in Norway. Deposit guarantee schemes (DGS) are set up with the purpose of providing depositors with the guarantee that their deposits will be repaid whenever a bank defaults. To perform this role, the DGS needs to have an adequate amount of funds at its disposal. This amount is usually set aside by collecting ex ante contributions from banks. According to Norwegian law, the Guarantee Fund must have administrative systems for calculating their possible future liabilities of paying out deposits. In this report, we describe a methodology to determine these liabilities. The results from the model can be used to evaluate the size of the deposit guarantee fund, the need for additional funding, and the distribution of annual contributions between the member banks.

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# Contents

<b>1</b>	<b>Introduction</b>	<b>5</b>
<b>2</b>	<b>Previous work</b>	<b>7</b>
2.1	Overall methodology	7
2.2	Probability of default	7
2.3	Loss-given-default	8
2.4	Correlations	9
<b>3</b>	<b>Total loss simulation</b>	<b>10</b>
3.1	Model	10
3.2	Simulation algorithm	11
3.2.1	Resolution	11
<b>4</b>	<b>Probability of default</b>	<b>13</b>
4.1	Risk-neutral probabilities	13
4.2	Mapping between risk-neutral and real-world probabilities	15
4.3	Which PD to use?	18
<b>5</b>	<b>Loss-given-default</b>	<b>19</b>
5.1	LGD for different capital instruments	19
5.2	Estimating the parameters of the beta distribution	21
5.3	Adjusted LGDs	22
5.4	Missing prices	23
5.5	Non-valid risk classes	24
5.6	LGD for the Norwegian deposit insurance fund	25
<b>6</b>	<b>Correlations</b>	<b>26</b>
6.1	Banks for which stock prices are available	26
6.2	Banks for which stock prices are not available	27
<b>7</b>	<b>Contribution calculations</b>	<b>29</b>
	<b>References</b>	<b>30</b>
<b>A</b>	<b>Capital structure in Norwegian banks</b>	<b>33</b>
<b>B</b>	<b>Asset correlations in the IRB approach</b>	<b>33</b>

# 1 Introduction

Deposit insurance schemes (DIS) are set up with the purpose of providing depositors with the guarantee that their deposits will be repaid whenever a bank defaults. To perform this role, the DIS needs to have an adequate amount of funds at its disposal. This amount is usually set aside by collecting ex ante contributions from banks. Many countries adjust the contributions to be paid according to the risk which is incurred by the DIS by guaranteeing the deposits in the respective banks.

The Norwegian Banks' Guarantee Fund is the deposit insurer in Norway. The Norwegian Financial Undertakings Act, Section 19-14 states that: *"The deposit guarantee scheme shall have in place appropriate systems to calculate its guarantee liability. The scheme shall at all times have available a deposit guarantee fund and other resources that are in reasonable proportion to its guarantee liability"*. The act further states in Section 19-10 that the Fund *"shall base the individual member's contribution [...] on the member's share of the deposit guarantee fund's overall guarantee liability"*. To meet these requirements, the Norwegian Bank's Guarantee Fund have designed a total liability model which is presented in this report. The model will fulfill three purposes. First, it will enable an assessment of the size of the fund which is available for payouts, and whether alternative funding is required. Second, the model may be used to assess each member bank's risk-based contribution to the fund. And third, it may be used to determine the investment strategy for the fund. If it is considered unlikely that the fund must be realized in the near future, it may be expedient for parts of the fund to follow a strategy which to a greater extent emphasizes return rather than low risk and liquidity.

IADI (2009) states that the majority of countries use their experience with bank failure losses to determine the target deposit insurance fund. Given sufficient data on previous losses, one may estimate the empirical distribution of losses and use that distribution to determine the level of losses the deposit fund should be able to absorb. However, countries like Norway, with very few failed banks will lack sufficient data to develop an accurate empirical loss distribution. Hence, instead the Credit Portfolio Approach has been used to model the target deposit insurance fund for many countries, included Columbia, Canada and Singapore (O'Keefe and Ufier, 2017). This approach to modelling the target deposit insurance fund is based on the loan portfolio model of Vasicek (1987, 2002), which is very similar to the portfolio model used in the advanced IRB approach of Basel II (Gordy, 2003). This model, which is also the one chosen for the Norwegian Banks' Guarantee Fund, assumes that obligors' asset value changes are determined by idiosyncratic and systemic risk factors. The systemic risk factor is common to all obligors and may be viewed as the current state of the economy.

The rest of this report is organized as follows. Chapter 2 gives an overview of previous work on estimating the size of the target deposit insurance fund, while Chapter 3 treats the model used here. This model requires estimates for the probability of default, loss-given-default and correlation parameter for each bank. In Chapters 4-6 we describe the approaches used to estimate these quantities. Finally, Chapter 7 describes how contributions from the individual member banks could be calculated based on the computed deposit guarantee fund liabilities.

## 2 Previous work

### 2.1 Overall methodology

Most papers constituting the academic literature on DIS (e.g. Cariboni et al. (2011); Garcia-Cespedes and Moreno (2014); Kuritzkes et al. (2002, 2005); Lee et al. (2015); O’Keefe and Ufier (2017)) use a variant of the IRB-model of Basel II to generate the loss distribution. In this approach, the DIS fund is regarded as a portfolio of counterparty risks. The portfolio consists of individual exposures to the insured banks, each having a small, but non-zero probability of default. The procedure used to simulate the loss distribution relies on classical credit risk techniques; defaults occur if the bank’s asset value falls below a threshold, where the asset values follows a Gaussian one-factor model.

The Systemic Model For Banking Originated Losses (SYMBOL) (Lisa et al., 2011; Zedda and Cannas, 2017) is a bit different from the above. In this approach, the average probability of default for the obligors in a bank’s credit portfolio is first obtained by numerical inversion of the Basel II IRB-formula, using the reported level of minimum capital requirement and total exposure, and setting the LGD, maturity and turnover to their standard values. Then, the loss distributions for all the banks are simulated using the Gaussian one-factor model, using the same total exposure, LGD, maturity and turnover as when determining the probability of default. Finally, for each bank and each scenario the difference between the simulated loss and the bank’s actual capital is computed. If the difference is negative, the bank is regarded to be insolvent in the specific simulation.

### 2.2 Probability of default

The approach for estimating the probability of default (PD) for each bank is different in different papers. First, there are public sources of risk ratings, such as from Standard & Poor’s or Moody’s. Both of these rating agencies give solvency standards for the rated institutions in the form of a credit grade, which may then be converted to a PD. Many banks are however not rated by the large rating agencies. For these banks CAMEL-style or shadow ratings may be used. These are unofficial ratings given to an issuing party by a credit agency (e.g. a bank), but without any public announcement of the rating. One potential disadvantage with these ratings is that they take expert judgement into account.

The next type of approaches for estimating the probability of default are logistic regression models (or other statistical models) where the dependent variable is 1 or 0 dependent on whether there is a bank failure or not during the time period of interest. The RiskCalc Banks model from Moody’s Analytics Inc. (Wang

et al., 2014) is a such model. The most informative explanatory variables will vary across countries. However, bank financial measures of capital adequacy, asset quality, management quality, earnings strength, liquidity and sensitivity to market prices are frequently listed as significant variables (O’Keefe and Ufier, 2017). Given the low frequency of bank failures, estimation of such models requires a long observation period, especially if macroeconomic indicators are used as explanatory variables.

A widely used insolvency risk measure in the banking and financial stability literature is the Z-score (Hannan and Hanweck, 1988). With this measure, bank insolvency is commonly defined as a state where the sum of the bank’s capital asset ratio and its return on assets is negative. As far as we are concerned, the Z-score has so far not been used in the deposit insurance fund literature. One reason for this might be that it is not evident how to link the Z-score to a probability of bank default. Traditionally, one has just linked the Z-score to an upper bound of the bank’s probability of insolvency given by  $1/(1 + Z\text{-score}^2)$ . However, in a very recent paper (Bouvatier et al., 2018), two alternative ways of converting Z-scores to probabilities are proposed.

Finally, the bank’s default probabilities may be estimated from CDS or bond market data, see e.g. Cariboni et al. (2011). In theory, marked-implied forecasts should be superior both to credit ratings and statistical models, since they may be updated in real time. The quality of market-based models depends however on the availability and quality of market data. Such models rely on the efficient financial markets hypothesis that assumes perfect liquidity, while in practice, the bond/CDS prices may include substantial liquidity premiums in addition to the credit risk premiums (Smirnov and Zdorovenin, 2012). Hence, risk neutral probabilities extracted from bond spreads can be considered as upper bounds for actual probabilities. It should also be noted that default rates inferred from the market data are risk-neutral, in contrast to real-world estimates produced by the alternative approaches. Hence, the risk-neutral probabilities should be mapped to actual ones before they are used to generate the DIS loss distribution.

## 2.3 Loss-given-default

The previous literature on Loss-Given-Default (LGD) is limited. Most papers treating DIS contain very little information on how the LGD values for the different banks are determined. In Cariboni et al. (2011) the LGD is e.g. set to 60% for all banks without any further explanation. Smirnov and Zdorovenin (2012) states that if available default history does not allow for meaningful LGD estimation, deposit insurers can adopt the foundation IRB approach of the Basel II accord. Here, a deposit insurer that has a senior claim on failed banks can assume a LGD



of 45%. Kuritzkes et al. (2005) and O’Keefe and Ufier (2017) note the fact that loss rates tend to decline as bank size increases and estimate a relationship between the LGD and the asset size of the banks.

Alternatively, one may use information embedded in credit default swaps or bond spreads to estimate LGD values. In Heynderickx et al. (2016a), a method originally proposed by Schlafer and Uhrig-Homburg (2010) is used to determine recovery rates of the European banking sector. The key factor of this method is that debt instruments of the same issuer with different rankings face identical default risk, but different default-conditional recovery rates. Using the ratio of a senior and a subordinated CDS and taking the specific liability structure of banks into account (which consists typically of hybrids, subordinated debt, senior unsecured debt, senior secured debt, deposits etc.), the recovery rate of senior debt may be computed.

## 2.4 Correlations

In all the papers using the Gaussian model one-factor model listed in Section 2.1, correlation between defaults of different banks is imposed through the parameter specifying the degree of dependence of the systematic factor. Like for the LGD, most papers treating DIS contain very little information on how this parameter is determined for different banks. In Cariboni et al. (2011) the parameter is set to 70% for all banks without any further explanation. O’Keefe and Ufier (2017) also use one value for all banks, but it is determined by first estimating pairwise Pearson correlations for each pair of bank stock returns (or returns on equity for non-listed banks) and then taking the average of these correlations.

If the available data does not allow for meaningful estimation of the correlations, they might be computed using the Basel II, IRB approach. Here, the correlation parameter decreases with increasing PD and increases with firm size. The intuition behind these relationships are as follows. The higher the PD, the higher the idiosyncratic (individual) risk components of a borrower. The default risk depends less on the overall state of the economy and more on individual risk drivers. The larger a firm, the higher its dependency upon the overall state of the economy, and vice versa. Smaller firms are more likely to default for idiosyncratic reasons.

### 3 Total loss simulation

We have chosen to follow Cariboni et al. (2011); Garcia-Cespedes and Moreno (2014); Kuritzkes et al. (2002, 2005); Lee et al. (2015); O’Keefe and Ufier (2017) and use a variant of the IRB-model of Basel II to generate the loss distribution. In this approach, the DIS fund is regarded as a portfolio of counterparty risks. The portfolio consists of individual exposures to the insured banks, each having a small, but non-zero probability of default. The procedure used to simulate the loss distribution relies on classical credit risk techniques; default occur if the bank’s asset value falls below a threshold, where the asset value follows a Gaussian one-factor model.

#### 3.1 Model

The total loss of the DIS a specific year  $t$  is given by

$$L_t = \sum_{n=1}^N w_{n,t} LGD_n D_{n,t},$$

where  $w_{n,t}$  and  $LGD_n$  are the exposure and loss-given-default, respectively, of bank  $n$ , and  $D_{n,t}$  is an indicator variable that is 1 if bank  $n$  defaults in year  $t$ . The exposure of a specific bank  $n$  is assumed to increase with a certain factor  $f_n$  for each year of the simulation period, i.e.

$$w_{n,t} = w_{n,0}(1 + f_n)^{t-1},$$

where  $w_{n,0}$  and  $f_n$  for all banks are input to the simulation module.

The probability of  $D_{n,t} = 1$  is  $p_{n,t}$ , where  $p_{n,t}$  is the probability of default (PD) of bank  $n$  in year  $t$ . In the Gaussian one-factor model one have that

$$D_{n,t} = 1 \Leftrightarrow a_{n,t} \leq \Phi^{-1}(p_{n,t}), \tag{1}$$

where

$$a_{n,t} = \sqrt{\rho_n} X_t + \sqrt{1 - \rho_n} \xi_{n,t}.$$

Here,  $X_t$  is the systematic risk factor and  $\xi_{n,t}$  the specific risk associated with bank  $n$  in year  $t$ .  $X_t$  and  $\xi_{n,t}$  are assumed to be independent and  $N(0,1)$ -distributed, and  $\rho_n$  is the correlation parameter associated with bank  $n$ .

With this model, the so-called asset correlation between two banks  $n$  and  $m$  is  $\text{Cor}(a_{n,t}, a_{m,t}) = \sqrt{\rho_n \rho_m}$ . Note, that the asset correlation is different from the default correlation, which is given by

$$\kappa_{n,m} = \frac{\Phi_2(\Phi^{-1}(p_n), \Phi^{-1}(p_m), \sqrt{\rho_n \rho_m}) - p_n p_m}{\sqrt{p_n(1-p_n)p_m(1-p_m)}}, \tag{2}$$

where  $\Phi_2(\cdot)$  is the cumulative bivariate normal distribution function. The default correlation usually is significantly smaller than the asset correlation.

When simulating losses on a  $T$ -year horizon, we allow for the systematic factor to follow a mean-reverting process in the following way:

$$X_t = \alpha X_{t-1} + \epsilon_t,$$

where  $\epsilon_t \sim N(0, 1 - \alpha^2)$ . If  $\alpha$  is zero, there is no dependence in time.

## 3.2 Simulation algorithm

The loss distribution for a specific year  $t$  is simulated using the following procedure:

1. Sample  $X_t = \alpha X_{t-1} + \epsilon_t$  where  $\epsilon_t \sim N(0, 1 - \alpha^2)$ .
2. For all banks  $n$ 
  - Sample  $\xi_{n,t} \sim N(0, 1)$ .
  - Compute  $a_n = \sqrt{\rho_n} X_t + \sqrt{1 - \rho_n} \xi_{n,t}$ .
  - Set  $D_{n,t} = 1$  if  $a_{n,t} \leq \Phi^{-1}(p_{n,t})$  and 0 otherwise.
  - Compute the loss for bank  $n$  as  $L_{n,t} = w_{n,t} LGD_n D_{n,t}$ .
3. Compute the total loss for year  $t$  as  $L_t = \sum_n L_{n,t}$ .

In addition to the total loss, The Norwegian Banks' Guarantee Fund is interested in the liquidity reserve. The liquidity reserve  $R_{n,t}$  for bank  $n$  in year  $t$  is given by

$$R_{n,t} = \begin{cases} 0 & \text{if } t < t_{def} \\ w_{n,t} & \text{if } t = t_{def} \\ w_{n,t} LGD_{n,t}^* & \text{if } t > t_{def} \end{cases}$$

Here,  $t_{def}$  is the year of default for bank  $n$  and  $LGD_{n,t}^*$  is computed as

$$LGD_{n,t}^* = LGD_{n,t-1}^* - b_{t-t_{def}}(LGD_{t_{def}}^* - LGD_n),$$

where  $LGD_{t_{def}}^*$  and  $b_1, b_2, b_3$  and  $b_4$  are input parameters to the simulation module. Usually  $LGD_{t_{def}}^*$  should be 1, and  $b_1, b_2, b_3$  and  $b_4$  should sum up to one, assuming that the final LGD,  $LGD_n$ , is known after 5 years.

The total liquidity reserve in year  $t$  is finally computed as  $R_t = \sum_n R_{n,t}$ .

### 3.2.1 Resolution

Resolution is the restructuring of a failing bank through the use of resolution tools in order to safeguard public interests. Some banks are categorized as too

systemically important and interconnected to allow for their liquidation through a normal insolvency process. For those banks the resolution tools can provide for an orderly wind-down of the bank or restore the viability of all or part of the institution. Through resolution the functions of the bank that are critical to the financial market or the real economy would be protected, while ensuring that losses are borne by shareholders and creditors of the failing bank.

Available financial means of the deposit guarantee scheme could be used in accordance to the resolution proceedings. The liability of the deposit guarantee scheme shall not be greater than the amount of losses that it would have had to bear had the institution been wound up under normal insolvency proceedings. The liability of a deposit guarantee scheme shall also not be greater than the amount equal to 50 % of the minimum target level of 0.8 % of total covered deposits. This means, that if bank  $n$  is critical to the financial market, its loss the year of default,  $t_{def}$ , is limited to

$$L_{n,t_{def}}^* = \min \left( w_{n,t_{def}} LGD_n, 0.5 \times 0.008 \times \text{Total covered deposits}_{t_{def}} \right),$$

where

$$\text{Total covered deposits}_{t_{def}} = \sum_n w_{n,t_{def}}.$$

Further, the liquidity reserve  $R_{n,t}$  for bank  $n$  in year  $t \geq t_{def}$  is limited to

$$R_{n,t}^* = \min \left( w_{n,t_{def}} LGD_n, 0.5 \times 0.008 \times \text{Total covered deposits}_t \right),$$

where

$$\text{Total covered deposits}_t = \sum_n w_{n,t}.$$

## 4 Probability of default

In a preliminary phase, all the alternatives discussed in Section 2.2 were considered. The rating-based approach was rejected due to the fact that most Norwegian banks are not rated by the large rating agencies. The logistic regression model can not be used given the very low frequency of historical bank failures, while the Z-score approach was not deemed appropriate due to volatile and non-intuitive results. Hence, we decided to use market data to estimate the probabilities of default. As stated in Section 2.2, marked-implied forecasts should in theory be superior to both credit ratings and statistical models. In Norway, CDS contracts are written only on a very limited number of banks. Hence, we have chosen to use bond yield spreads instead. The specific method used to derive the risk neutral probabilities is described in Section 4.1. The risk-neutral probabilities should be mapped to actual ones before they are used to generate the DIS loss distribution. In Section 4.2 we describe the conversion method used in this project. The actual default probabilities are computed for all time points represented in the data set used for estimation giving rise to a PD time series for each bank. In Section 4.3 we describe how obtain the final PDs that are to be used in the loss simulations from these time series'.

### 4.1 Risk-neutral probabilities

The bond spread data used to estimate the risk-neutral probabilities of default consists of prices of senior unsecured, subordinated and hybrid securities. Different financial instruments from the same bank are in theory exposed to the same default risk. We have therefore chosen to determine the risk-neutral probability of default for a specific bank as a function of a hazard rate computed as an average over the hazard rates of three different types of instruments. In what follows we describe the procedure for determining the probabilities in more detail.

Let the probability of default between horizon  $h$  and horizon  $h + 1$  for a specific instrument be given by

$$P(h, h + 1) = 1 - \exp\{-\lambda(h, h + 1)\}, \quad (3)$$

where  $\lambda(h, h + 1)$  is the average hazard rate or default intensity during the time period between horizon  $h$  and horizon  $h + 1$ . It is given by

$$\lambda(h, h + 1) = (h + 1) \lambda(0, h + 1) - h \lambda(0, h). \quad (4)$$

where  $\lambda(0, h)$  is the average hazard rate during the time period between year 0 and year  $h$ <sup>1</sup>.

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1. The probability of default between time 0 and time  $h$  is given by  $P(0, h) = 1 - \exp\{-\lambda(0, h)h\}$ .

Suppose now that  $s(h)$  is the bond yield spread for a  $h$ -year bond. Then, it is approximately true (Hull, 2015) that

$$\lambda(0, h) = \frac{s(h)}{1 - R}, \quad (5)$$

where  $R$  is the estimated recovery rate for the actual bond.

Corporate bond spreads are affected by both credit and liquidity risk. When using the formula in Equation 5, one should ideally use only the fraction of the spreads driven by the credit risk. Many researchers have tried to disentangle the separate effects of credit and liquidity risk on corporate bond yields, see e.g. Collin-Dufresne et al. (2001); Driessen (2005); Helwege et al. (2014); Longstaff et al. (2005). However, this is very challenging, due to the fact that liquidity risk isn't readily measured. In some studies, the credit risk is shown to account for only a small fraction of the observed corporate bond spreads, while in other, the portion of the spread driven by credit risk is claimed to be much larger. Moreover, since the research conducted up to date almost exclusively has focused on the U.S. or large European markets, little is known about the liquidity of the Norwegian corporate bond market. We have therefore decided to let the fraction of the bond spreads accounted for by the credit risk be an input to the estimation module. More specifically, the user specifies a matrix like the one below, where the number in a specific cell gives the fraction of the corresponding bond spread accounted for by credit risk.

Instr./h	1y	2y	3y	4y	5y
Senior	0.9	0.7	0.7	0.7	0.6
Hybrid	0.9	0.7	0.7	0.7	0.6
Sub	0.9	0.7	0.7	0.7	0.5

Before using Equation 5 to compute the hazard rates, the bond spreads  $s(h)$  are reduced according to the specified fractions. The yearly default probabilities are then derived as follows:

1. We compute hazard rates  $\lambda(0, h)$  for horizons  $h = 1, \dots, 6$  years using Equation 5.
2. We obtain yearly hazard rates  $\lambda(h, h + 1)$  for year  $h = 1, \dots, 5$  using Equation 4<sup>2</sup>.
3. We compute one year default probabilities using Equation 3.

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2. Note that if the adjustment factors for the hazard rates  $\lambda(0, h)$  and  $\lambda(0, h + 1)$  are very different, one might get negative yearly hazard rates  $\lambda(h, h + 1)$ . If this is the case, the yearly hazard rate is set to a positive tiny number.

As described above, different financial instruments from the same bank are in theory exposed to the same default risk. For a specific bank, step 1 above is therefore replaced by the following (note that  $LGD = 1-R$ )<sup>3</sup>:

- Compute  $\lambda_{senior}(0, h) = s_{senior}(h)/LGD_{senior}(5)$ .
- Compute  $\lambda_{sub}(0, h) = s_{sub}(h)/LGD_{sub}(5)$ .
- Compute  $\lambda_{hybrid}(0, h) = s_{hybrid}(h)/LGD_{hybrid}(5)$ .
- Compute  $\lambda_{avg}(0, h) = w_{senior} \lambda_{senior}(0, h) + w_{sub} \lambda_{sub}(0, h) + w_{hybrid} \lambda_{hybrid}(0, h)$ .

The weights  $w_{senior}$ ,  $w_{sub}$  and  $w_{hybrid}$  are input to the program. The framework for determining  $LGD_{senior}(5)$ ,  $LGD_{sub}(5)$  and  $LGD_{hybrid}(5)$  is given in Section 5.1. Since it is common to keep the risk neutral LGD fixed over the different durations for a given level of seniority, we use the 5-years LGD for all values of  $h$ . The choice of the 5-year LGD, is due to the fact that the corresponding instruments are usually regarded to be the most liquid ones.

## 4.2 Mapping between risk-neutral and real-world probabilities

Let  $PD_{rn}$  be the risk-neutral default probability for a specific bank and specific year computed as described in Section 4.1 (the subscripts are here dropped for simplicity). To determine the mapping between risk-neutral and actual default probabilities we use the approach proposed in Heynderickx et al. (2016b). Here, the relationship between the coverage ratio  $\mu$  defined as

$$\mu = \frac{PD_{rn}}{PD_{rw}},$$

and the actual default probabilities  $PD_{rw}$  is modelled by

$$\log(\mu) = a (PD_{rw})^b, \tag{6}$$

where the parameters  $a$  and  $b$  are input to the estimation module of the software. When using Equation 6 to compute the actual default probabilities, the risk-neutral probabilities might not be a strictly increasing function  $f(\cdot)$  of the default probabilities. To ensure that this function is monotonically increasing, we set  $PD_{rw}$  equal to  $(-1/(a \cdot b))^{1/b}$  if it is lower than this value, which is the minimum value of the function  $f(\cdot)$ .

In Heynderickx et al. (2016b), the parameters  $a$  and  $b$  were estimated based on a sample of around 550 European private sector companies, including banks, other

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3. Note that there are no market prices available for the horizon  $h = 4$ . Hence, these prices are obtained as an average of the prices for  $h = 3$  and  $h = 5$ .

financial and non-financial corporates. The actual default probabilities were estimated from transition matrices computed by rating agencies, while the risk-neutral probabilities were derived from CDS data. Different values of  $a$  and  $b$  were estimated for different predefined sub-periods. The results are reproduced in Table 1.

Table 1. Parameter values from Heynderickx et al. (2016b).

<i>Period</i>	<i>Duration</i>	<i>a</i>	<i>b</i>	<i>R<sup>2</sup>-adj</i>
Entire period	2004-2014	4.1649	-0.2588	0.6162
Pre-crisis	2004-2007	2.9242	-0.2975	0.5036
Financial crisis	2008-2009	4.7708	-0.2460	0.8671
Sovereign crisis	2010-2012	5.1012	-0.2263	0.9126
Post-crisis	2013-2014	4.7200	-0.3015	0.8276

In a preliminary phase of this project we also estimated  $a$  and  $b$  using shadow ratings from SpareBank 1 Markets from Q1, 2018 and bond spread data from Nordic Bond pricing from June 29th, 2017. The shadow ratings were converted to actual default probabilities according to the relationships shown in Table 2. The estimated relationship between the coverage ratio and the actual default probabilities is shown in Figure 1, corresponding to  $a = 6.92$  and  $b = -0.33$ .



Table 2. Relationship between ratings and default probabilities (source: Moody's).

<i>Rating</i>	<i>PD</i>
AAA	0.002%
AA+	0.020%
AA	0.035%
AA-	0.060%
A+	0.090%
A	0.150%
A-	0.240%
BBB+	0.380%
BBB	0.620%
BBB-	1.000%
BB+	1.620%
BB	2.620%
BB-	4.240%
B+	6.850%
B	11.090%
B-	17.940%
CCC+	29.030%
CCC	46.980%
CCC-	76.010%

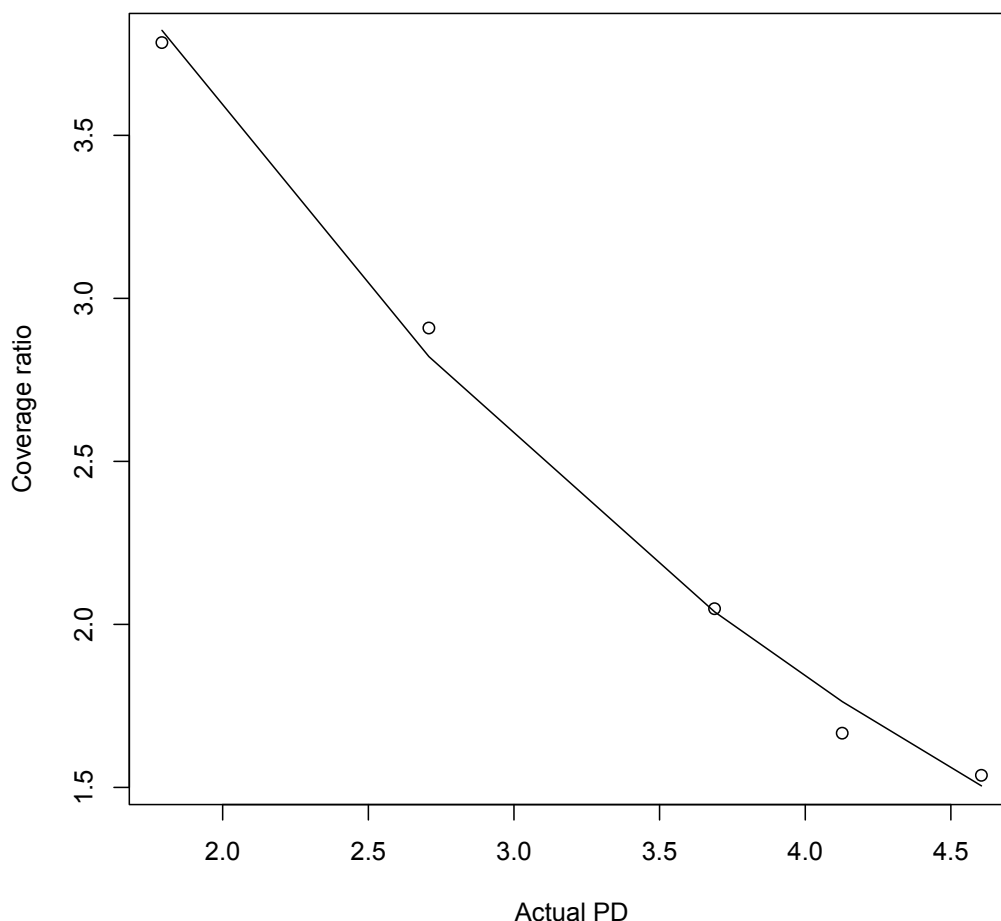


Figure 1. The relationship between the coverage ratio and the actual default probabilities.

### 4.3 Which PD to use?

The real-world PDs resulting from the procedures described in Sections 4.1 and 4.2 are first computed for all time points represented in the data set with historical prices given as input to the software. Then, the final PDs that are to be used in the loss simulations are determined using one of the following three approaches:

- The final PD for a specific bank and a specific horizon are the one computed for the last time point of the input data.
- The final PD for a specific bank and a specific horizon is computed as the average over all available PDs for this bank/horizon.
- The final PD for a specific bank and a specific horizon is computed as the maximum over all available PDs for this bank/horizon.

## 5 Loss-given-default

Like for the probability of default, we have chosen to use information embedded in bond spreads to estimate loss-given-default (LGD) values. Our approach is based on the work described in Heynderickx et al. (2016a). We first fit a model for the firm-wide recovery rate valid for all banks. Then, for each specific bank  $n$ , we use this model and the capital structure of the bank to determine instrument specific LGDs. These LGDs are later adjusted before they are used to compute the deposit insurance fund loss-given-default for bank  $n$ .

More specifically, the procedure for obtaining the LGD values is as follows:

1. Build a model for estimating the parameters of a beta distribution used to determine LGDs with 5-years maturity for different capital instruments
2. For each bank  $n$ 
  - a. Use the model from step 1 to determine  $LGD_{bank}(n, 5)$ ,  $LGD_{senior}(n, 5)$ ,  $LGD_{sub}(n, 5)$  and  $LGD_{hybrid}(n, 5)$
  - b. Compute adjusted values  $LGD_{senior}^{adjust}(n, 5)$ ,  $LGD_{sub}^{adjust}(n, 5)$ ,  $LGD_{hybrid}^{adjust}(n, 5)$ .
  - c. Determine  $LGD_{DIF}(n)$  from  $LGD_{bank}(n, 5)$ ,  $LGD_{senior}^{adjust}(n, 5)$ ,  $LGD_{sub}^{adjust}(n, 5)$ , and  $LGD_{hybrid}^{adjust}(n, 5)$ .

In the above procedure,  $LGD_{senior}(n, 5)$ ,  $LGD_{sub}(n, 5)$  and  $LGD_{hybrid}(n, 5)$  are the estimated LGDs for senior unsecured, subordinated and hybrid securities for bank  $n$  at 5-years maturity, while  $LGD_{bank}(n, 5)$  is the firm-wide loss-given default for the same maturity. Further,  $LGD_{senior}^{adjust}(n, 5)$ ,  $LGD_{sub}^{adjust}(n, 5)$ , and  $LGD_{hybrid}^{adjust}(n, 5)$  are adjusted versions of  $LGD_{senior}(n, 5)$ ,  $LGD_{sub}(n, 5)$  and  $LGD_{hybrid}(n, 5)$ , and finally,  $LGD_{DIF}(n)$  is the deposit insurance fund loss-given-default for bank  $n$ .

In Sections 5.1 and 5.2 steps 2a) and 1) are more thoroughly described, while the approach for computing the adjusted LGDs in step 2b) is outlined in Section 5.3. Finally, Section 5.6 contains the procedure for obtaining the deposit insurance fund loss-given-default in step 2c).

### 5.1 LGD for different capital instruments

The LGD for a specific instrument  $instr$  issued by a bank  $n$  at time  $t$ , is given by

$$LGD_{instr,n,t} = 1 - E[R_{instr,n,t}]$$

If the absolute priority rule (APR) holds, recovery rates are a function only of the ratio of firm value to total liabilities and the capital structure, both at time

of default. Hence, we assume that the expected risk neutral recovery rate for a specific instrument  $instr$  given by (Heynderickx et al., 2016a)

$$E[R_{instr,n,t}] = \int_0^1 \rho_{instr,n,t}(x) h_{n,t}(x) dx, \quad (7)$$

where  $x$  is the ratio of the value of a bank at default and its liabilities,  $\rho_{instr,n,t}(x)$  is the instrument-specific recovery rate, and  $h_{n,t}(x)$  is the risk neutral density of recovery. The expected firm-wide recovery rate, which is equal to the expected ratio of the firm value to liabilities at default, is given by

$$E[R_{firm,n,t}] = \int_0^1 x h_{n,t}(x) dx. \quad (8)$$

The risk-neutral density of recovery is assumed to be a beta distribution (this is a common assumption, see e.g. Gupton and Stein (2002)). Past studies suggest that the observed recovery can be explained by some firm specific factors (Heynderickx et al., 2016a; Schlafer and Uhrig-Homburg, 2010). We therefore let the expectation of the beta distribution<sup>4</sup> be given by

$$\text{logit}(\mu_{n,t}) = \gamma_0 + \gamma_1 \log(C_{n,t}),$$

where  $C_{n,t}$  is the total assets for bank  $n$  at time  $t$ . The maximum possible variance of the beta distribution depends on the mean of the distribution. Like Heynderickx et al. (2016a) we therefore express the standard deviation as a function of the average recovery rate, i.e.

$$\sigma_{n,t} = \gamma_2 \sqrt{\mu_{n,t} - \mu_{n,t}^2}.$$

The parameters  $\gamma_0$ ,  $\gamma_1$  and  $\gamma_2$  are estimated from historical bond spreads. The estimation procedure is described in Section 5.2.

Let the proportions of hybrids, subordinated debt, senior unsecured debt and a rest for a specific bank at a specific point in time be  $P_h$ ,  $P_j$ ,  $P_s$  and  $P_l$ , respectively, with  $P_l + P_s + P_j + P_h = 1$ . See Appendix A for the definitions of the different types of debt in our case. The recovery rates for senior unsecured, subordinate and hybrid debt are given by (for the sake of simplicity we here omit the subscripts  $n$  and  $t$ ):

$$\rho_{senior}(x) = \begin{cases} 0 & x \in [0, P_l] \\ \frac{x - P_l}{P_s} & x \in [P_l, P_l + P_s] \\ 1 & x \in [P_l + P_s, 1] \end{cases},$$

$$\rho_{sub}(x) = \begin{cases} 0 & x \in [0, P_l + P_s] \\ \frac{x - (P_l + P_s)}{P_j} & x \in [P_l + P_s, 1 - P_h] \\ 1 & x \in [1 - P_h, 1] \end{cases},$$

---

4. The beta distribution has two parameters  $\alpha$  and  $\beta$ , and the mean and variance of the distribution are given by  $\mu = \frac{\alpha}{\alpha + \beta}$  and  $\sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$ , respectively.

and

$$\rho_{\text{hybrid}}(x) = \begin{cases} 0 & x \in [0, 1 - P_h] \\ \frac{x - (1 - P_h)}{P_h} & x \in [1 - P_h, 1] \end{cases}.$$

## 5.2 Estimating the parameters of the beta distribution

To determine the parameters of the beta distribution in Section 5.1, we use the approach proposed in Heynderickx et al. (2016a), which again is based on the work of Schlafer and Uhrig-Homburg (2010). This approach exploits the pricing information from two different financial instruments which are exposed to the same default risk, but have different recovery rates. Given this information and an assumption regarding the (in)dependence between recovery and default rates, one can estimate the distribution of the risk-neutral recovery.

As previously stated, the liability structure of a bank typically consists of hybrids, subordinated debt, senior unsecured debt and a rest. Our method utilises the fact that for a specific bank, different financial instruments have different recovery rates, but they are exposed to the same default risk. Hence, assuming that  $\lambda_{\text{senior}} = \lambda_{\text{sub}} = \lambda_{\text{hybrid}}$  for a given date  $t$ , bank  $n$  and maturity/horizon  $h$  we have from Equation 5 that

$$\frac{LGD_{\text{senior}}}{LGD_{\text{sub}}} = \frac{s_{\text{senior}}}{s_{\text{sub}}} = \frac{1 - E[R_{\text{senior}}]}{1 - E[R_{\text{sub}}]},$$

and

$$\frac{LGD_{\text{senior}}}{LGD_{\text{hybrid}}} = \frac{s_{\text{senior}}}{s_{\text{hybrid}}} = \frac{1 - E[R_{\text{senior}}]}{1 - E[R_{\text{hybrid}}]},$$

where  $s_{\text{senior}}$ ,  $s_{\text{sub}}$  and  $s_{\text{hybrid}}$  are the prices of respectively a senior unsecured, subordinated and hybrid security, and  $E[R_{\text{senior}}]$ ,  $E[R_{\text{sub}}]$  and  $E[R_{\text{hybrid}}]$  the corresponding theoretical expected recovery rates. Note that the prices, and hence the recovery and LGD-values are different for different dates  $t$ , different banks  $n$ , and different maturities/horizons  $h$ , but to simplify the notation we have removed the subscripts here.

The above relationships mean that we can estimate the parameters  $\gamma_0$ ,  $\gamma_1$  and  $\gamma_2$  in the beta distribution by calibrating model-implied ratios to the actual ones. More specifically, we minimize

$$\sqrt{\frac{1}{N} \frac{1}{T_N} \sum_{n=1}^N \sum_{t=1}^{T_n} [(V_{\text{model},A}(t, n) - V_{\text{obs},A}(t, n))^2 + (V_{\text{model},B}(t, n) - V_{\text{obs},B}(t, n))^2]}, \quad (9)$$

with respect to the parameters  $\gamma_0$ ,  $\gamma_1$  and  $\gamma_2$ , respectively, or just  $\gamma_0$  and  $\gamma_2$  if the user of the software does not want the beta distribution to be bank specific<sup>5</sup>. In Equation 9,  $N$  is the number of banks for which prices are available and  $T_n$  is the number of quarterly observations for bank  $n$ . Further

$$\begin{aligned} V_{model,A} &= (1 - E[R_{senior}]) / (1 - E[R_{sub}]), \\ V_{model,B} &= (1 - E[R_{senior}]) / (1 - E[R_{hybrid}]), \\ V_{obs,A} &= s_{senior} / s_{sub}, \\ V_{obs,B} &= s_{senior} / s_{hybrid}, \end{aligned}$$

where  $E[R_{instr,n,t}]$  is given by Equation 7. Note that this optimization is performed using securities with maturity/horizon  $h = 5$  years, only. This is due to the fact that we have decided to use 5-years LGDs in the simulation algorithm described in Section 3.2. Note also that when performing the minimization in Equation 9, we only use observations for which all of  $s_{senior}$ ,  $s_{sub}$  and  $s_{hybrid}$  exist. Finally, one should be aware of the fact that if the 5-year bond spreads  $s_{senior}$ ,  $s_{sub}$  and  $s_{hybrid}$  are adjusted with *different* factors to account for different liquidity premiums (see Section 4.1), the ratios  $V_{obs,A}$  and  $V_{obs,B}$  will change. This means that the parameters of the beta distribution also might be different from those obtained when using the original bond spreads.

### 5.3 Adjusted LGDs

As stated in Section 5.1, for a specific bank, different financial instruments have different recovery rates, but they are exposed to the same default risk. Hence, if we derive the risk neutral default probabilities for the hybrids, subordinated deb and senior unsecured debt for a specific bank using the LGDs estimated in Section 5.1, these default probabilities should in theory be equal. However, in real life, this will seldom be true. Hence, we have decided to compute adjusted LGD values using the procedure outlined below, where we use average hazard rates computed as described in Section 4.1. Note that all quantities are computed for a specific bank  $n$  at a specific time point  $t$ , but to simplify the notation we have omitted the subscripts  $t$  and  $n$ .

- Determine  $LGD_{senior}^{adjust}(5) = \frac{s_{senior}(5)}{\lambda_{avg}(0,5)}$
- Determine  $LGD_{sub}^{adjust}(5) = \frac{s_{sub}(5)}{\lambda_{avg}(0,5)}$

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5. We use numerical optimization when minimizing Equation 9. This means that we need to specify start values for the parameters. In the current version of the software, the start values of  $\gamma_0$ ,  $\gamma_1$  and  $\gamma_2$  are set to respectively 2, 0.1, and 0.37, if the beta distribution is to be bank specific, while the start values of  $\gamma_0$  and  $\gamma_2$  are set to 6 and 0.5 if the beta distribution is not to be bank specific.

- Determine  $LGD_{hybrid}^{adjust}(5) = \frac{s_{hybrid}(5)}{\lambda_{avg}(0,5)}$

Here,  $s_{senior}(5)$ ,  $s_{sub}(5)$  and  $s_{hybrid}(5)$  are the prices for senior, subordinated and hybrid securities at 5-years maturity, while  $\lambda_{avg}(0, 5)$  is the average hazard rate.

## 5.4 Missing prices

As previously mentioned, when estimating the parameters of the beta distribution using the procedure described in Section 5.2, all observations for which there are missing prices are removed. However, when computing the risk-neutral probabilities in Section 4.1 and the adjusted LGDs in Section 5.3 for all banks at all points in time, we need to replace any missing prices. To replace the missing prices, we have to estimate the corresponding risk classes. The procedure for estimating the risk class of an instrument, for which the risk class is not known at a certain point in time, is performed in two rounds. In the first round, for each time point and each bank we use the following rules:

- If the risk classes of two of the instruments are known, the third instrument will be assigned the risk class of the instrument just above in the debt hierarchy. Hence, if the risk class of hybrid is missing, it will be given the risk class of subordinated, subordinated will be given the risk class of senior unsecured, while senior unsecured will be given the risk class of subordinated, since senior unsecured is at the top of the debt hierarchy.
- If the risk class of only one of the instruments is known, the risk class of this instrument is used also for the two other instruments.
- If the risk classes of all three instruments are missing, nothing is done in this round.

There will be observations for which the risk class assigned using this procedure has no corresponding price. An example is shown in Table 3. The table shows for a certain point in time, and each risk class, the instruments for which prices exist. Assume that a bank has risk class 15 for subordinated and hybrid securities. Then, according to the procedure above, we will assign risk class 15 for senior securities also. However, at this point in time there are no prices for senior securities with risk class 15. In such cases we select the first risk class moving upwards in the table which have available prices. In this example this would be risk class 12.

The result of the first round, will be a data set for which either all, or none of the instruments have a risk class assigned. In the second round, we go through this data set, and for each instrument of a bank missing the risk class, we use the last observed risk class for this instrument and this bank. If there for a bank are no previous risk class observations, the risk class is set to  $c$  for all instruments and

Instruments for which prices are available	Risk Class
Sen, Sub, Hyb	1
Sen, Sub, Hyb	2
Sen, Sub, Hyb	3
Sen, Sub, Hyb	4
Sen, Sub, Hyb	5
Sen, Sub, Hyb	6
Sen, Sub, Hyb	7
Sen, Sub, Hyb	8
Sen, Sub, Hyb	9
Sen, Sub, Hyb	10
Sen, Sub, Hyb	11
Sen, Sub, Hyb	12
Sub, Hyb	13
Hyb	14
Sub, Hyb	15

Table 3. A list over instruments for which prices are available. Sen stands for Senior Unsecured, Sub for Subordinated and Hyb for Hybrid

all time points. Here  $c$  is a parameter which is input to the software.

## 5.5 Non-valid risk classes

The method for filling missing prices described in Section 5.4 assumes an ordering of the risk classes, as shown in Table 3. Some banks have a special risk class that does not fit into this hierarchy. Two examples are Bank Norwegian and Pareto Bank, where the risk class is simply the name of the bank. We therefore replace the risk class with a risk class that fits in the risk hierarchy using the following procedure for each instrument:

- If the price is lower than the price of risk class 1, assign risk class 1.
- If the price is between two risk classes, assign the risk class of the closest price.
- If the price is higher than the price of the highest risk class, assign the highest risk class plus 1, i.e. create a new risk class.

Note that we are only changing the risk class for banks with non-valid risk classes. The price is kept the same and no information is lost. This is only to make sure the procedure described in Chapter 5.4 works as expected.



## 5.6 LGD for the Norwegian deposit insurance fund

Given that we have computed the adjusted LGDs as described in Section 5.3, the final step is to determine  $LGD_{DIF}(n)$  the Norwegian deposit insurance fund LGD for bank  $n$ . This LGD is computed using the following procedure:

First we determine the potential losses connected to the different capital instruments:

$$\text{Hybrid loss}_n = LGD_{hybrid}^{adjust}(n, 5) \cdot \text{Hybrid capital}_n$$

$$\text{Subordinate Loss}_n = LGD_{sub}^{adjust}(n, 5) \cdot \text{Subordinate capital}_n$$

$$\text{Senior loss}_n = LGD_{senior}^{adjust}(n, 5) \cdot \text{Senior capital}_n$$

$$\text{Capital debt instrument loss}_n = \text{Hybrid loss}_n + \text{Subordinate Loss}_n + \text{Senior loss}_n.$$

Then, we compute the following quantities

$$\text{Total debt loss}_n = LGD_{bank}(n, 5) \cdot \text{Total liabilities}_n$$

$$\text{Non-guaranteed loss}_n = c_1 \cdot LGD_{k_1}(n, 5) \cdot (\text{Total deposits}_n - \text{Guaranteed deposits}_n)$$

$$\text{Other debt loss}_n = c_2 \cdot LGD_{k_2}(n, 5) \cdot \text{Other debt}_n$$

$$\begin{aligned} \text{Loss before covered deposits}_n &= \text{Capital instrument loss}_n + \text{Non-guaranteed loss}_n \\ &+ \text{Other debt loss}_n, \end{aligned}$$

where  $c_1$  and  $c_2$  are two parameters that are specified by the user. The parameters  $k_1$  and  $k_2$  can be set to *senior* or *bank* by the user.

Finally, the Norwegian deposit insurance fund LGD is determined by

$$LGD_{DIF}(n) = \max \left( c_3, \frac{\text{Total loss}_n - \text{Loss before covered deposits}_n}{\text{Guaranteed deposits}_n} \right),$$

where  $c_3$  is a minimum limit which is specified by the user.

## 6 Correlations

As previously stated, using the Gaussian model one-factor model, correlation between defaults of different banks is imposed through the parameter  $\rho$  specifying the degree of dependence of the systematic factor. Like O’Keefe and Ufier (2017), we use historical stock returns to determine the  $\rho$ -values for the banks that are listed on the Norwegian Stock Exchange. The estimation procedure is described in Section 6.1. For non-listed banks have to use an alternative approach, see Section 6.2.

### 6.1 Banks for which stock prices are available

O’Keefe and Ufier (2017) determine asset correlations by estimating pairwise Pearson correlations for each pair of bank stock returns. Since the number of member banks in the Norwegian Banks’ Guarantee Fund is quite large, a such procedure would mean that we have to estimate a huge correlation matrix. Hence, instead we have chosen to estimate the asset correlation for a specific bank by the correlation between the bank stock price returns and the returns of the total stock index (OSEBX). That is, the total stock index is used as a proxy for the systematic factor in the Gaussian one-factor model.

The correlations between the bank stocks and the total stock index varies over time. The length of the data series used to estimate the correlations plays a significant role. A shorter window is more reflective of the economic environment, but limits the number of data points, which ultimately produces more unstable and noisy estimates.

Understanding how the asset correlation change through time will allow us to investigate how the dynamics of this parameter behave during periods of economic stress. To explore the time-varying dynamics of the asset correlations, we follow a moving window approach. We use a window of length 2 years with monthly resolution to compute the first asset correlation. Next, we move the window forward by one month, and compute a new estimate, and so on. In Figure 2 we have computed the moving window correlations during the period October 1998 to October 2018 for 16 different stocks. As can be seen from the figure, the correlations change quite much through time. From the conservative perspective, one should use a high quantile in the shown time series as the estimate for the asset correlation in the loss simulations.

We have chosen to let the user decide whether he wants to use median correlations or extreme correlations as input to the loss simulations. With extreme correlations, we here mean a certain quantile  $q$  of the moving window correlation time series, where  $q$  is input to the program.

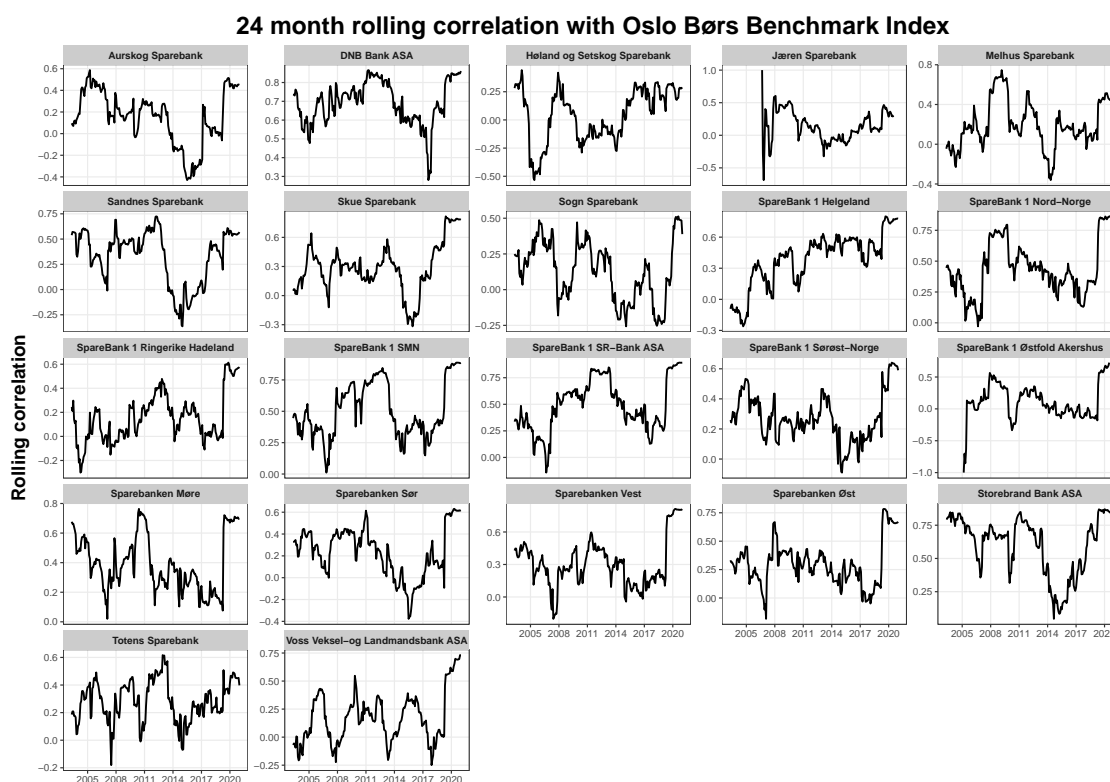


Figure 2. Time-varying dynamics of the asset correlations.

## 6.2 Banks for which stock prices are not available

For non-listed banks we cannot use the approach described in Section 6.1. For these banks, and for banks for which the historical time series is limited<sup>6</sup>, we instead estimate a model relating the median correlation to the total capital of the bank. We want the correlation to be in the interval (0,1). Hence, we use beta regression (Ferrari and Cribari-Neto, 2004) instead of ordinary linear regression. In this method the response is beta-distributed and related to other variables through a regression structure. We estimate the beta regression model using data for the banks for which we have more than 10 years of monthly data. Figure 3 shows the estimated relationship between the total capital of the bank and the median correlation with the total stock index with the actual observations superimposed.

The extreme correlations for non-listed banks are also determined by regression, but in this case we assume a linear model with the extreme and median correlations as response and explanatory variable, respectively. Figure 4 shows the estimated relationship between the median correlation and the extreme correla-

6. In the current version of the software, the approach described in Section 6.1 is used only for banks for which we have more than 10 years of monthly data.

tion with the actual observations superimposed. In this specific case, the extreme correlation is determined as the 90% quantile of the moving window correlation time series.

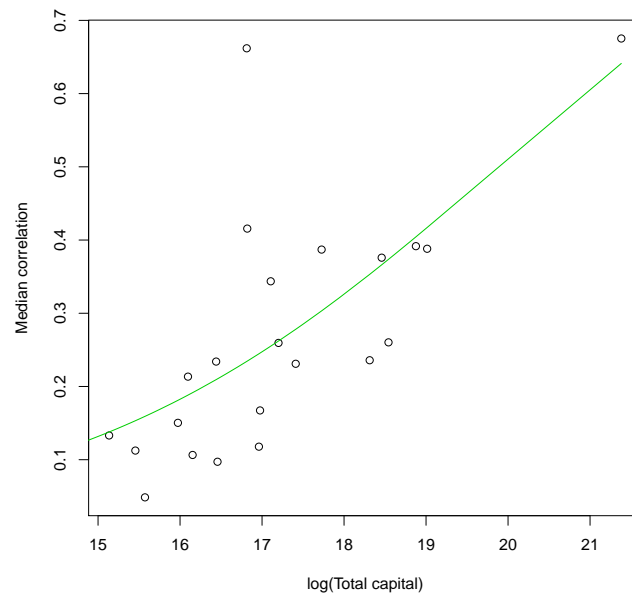


Figure 3. Relationship between total assets and median correlation. The green line shows the relationship estimated by the beta regression.

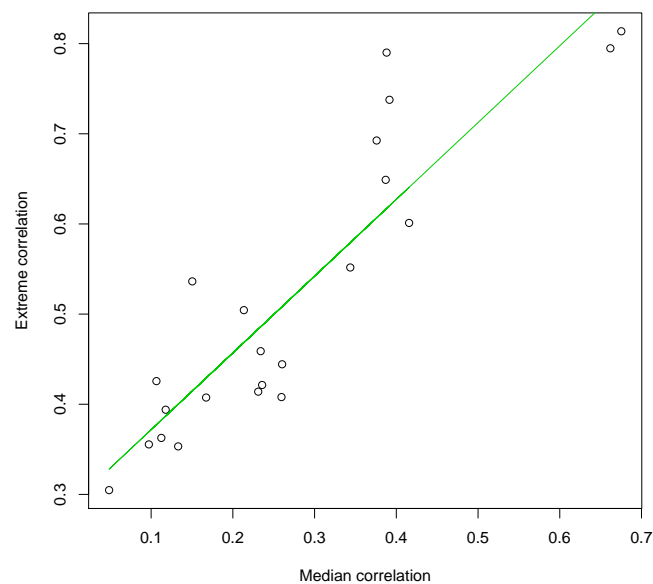


Figure 4. Relationship between median correlation and extreme correlation. The green line shows the relationship estimated by the linear regression.

## 7 Contribution calculations

Having computed the deposit guarantee fund liabilities using the method described in Chapter 3, the final step is to calculate the contributions of the member institutions. This can be done in many ways, see e.g. Kuritzkes et al. (2002), Huang et al. (2012) and Zedda and Cannas (2017). We have chosen to use the following approach to determine the contribution of bank  $n$  in year  $t$ .

Let  $L_{n,t}$  and  $L_t$  be the loss for bank  $n$  and the total loss over all banks during the period  $s = 1, \dots, t$ , respectively. Assume that the size of the total deposit fund is set to the  $q$ -percent quantile of the total loss, i.e.  $\text{VaR}_q(L_t)$ . Then, the absolute and relative contribution of bank  $n$  is computed as

$$C_{n,t} = \kappa_{n,t} \text{VaR}_q(L_t), \quad (10)$$

and

$$\kappa_{n,t} = \frac{\text{E}[L_{n,t} | L_t > \text{VaR}_q(L_t)]}{\text{E}[L_t | L_t > \text{VaR}_q(L_t)]}. \quad (11)$$

When  $\kappa_{n,t}$  is defined in this way, we are ensured that

$$\sum_{n=1}^N C_{n,t} = \text{VaR}_q(L_t).$$

If the contributions alternatively are to be calculated based on the liquidity reserve, we use exactly the same approach, but  $L_{n,t}$  and  $L_t$  are replaced with  $R_{n,t}$  and  $R_t$ , respectively.

As an alternative to the approach above, the contributions may also be computed using mean values of the losses and the liquidity reserves. In this case, the absolute and relative contributions instead are given by

$$C_{n,t} = \text{E}[L_{n,t}], \quad (12)$$

$$\kappa_{n,t} = \frac{\text{E}[L_{n,t}]}{\text{E}[L_t]}. \quad (13)$$

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## A Capital structure in Norwegian banks

- Senior unsecured = Gjeld stiftet ved utstedelse av verdipapirer
- Subordinate =  $(1-a) \cdot (\text{Fondsobligasjoner og ansvarlig lånekapital} + \text{Hybridkapital klassifisert som egenkapital})$
- Hybrid =  $a \cdot (\text{Fondsobligasjoner og ansvarlig lånekapital} + \text{Hybridkapital klassifisert som egenkapital})$
- Rest = Sum gjeld - Gjeld stiftet ved utstedelse av verdipapirer - Fondsobligasjoner og ansvarlig lånekapital

der

$$a = \frac{\text{Kjernekapitaldekning} - \text{ren kjernekapitaldekning}}{\text{Kapitaldekning} - \text{ren kjernekapitaldekning}}.$$

Totale forpliktelser = Sum gjeld + Hybridkapital klassifisert som egenkapital

## B Asset correlations in the IRB approach

The Basel Committee has provided different formulas for the asset correlations for different business segments (IRB-types). For large corporate borrowers including banks the asset correlation is computed by

$$\rho = 0.12 \times \frac{1 - e^{-50p}}{1 - e^{-50}} + 0.24 \times \left( 1 - \frac{1 - e^{-50p}}{1 - e^{-50}} \right), \quad (\text{B.1})$$

where  $p$  is the probability of default for the specific borrower.